Manifestation of $s \to \Lambda$ fragmentation matrix elements via transverse Λ polarization in unpolarized $e^- e^+$ annihilation

Wei Lu
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
*and Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China**

Xueqian Li
CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Department of Physics, Nankai University, Tianjin 300071, China

Haiming Hu
Department of Physics, Peking University, Beijing 100871, 300071, China

Making use of the collinear expansion technique developed by Ellis, Furmanski and Petrozio, and the special propagator concept invented by Qiu, we present a factorization approach to the photon fragmentation tensor for the inclusive Lambda hyperon production. As a result, the structure function $\tilde{F}$, which is related to the transverse polarization of the inclusive Lambda hyperon in unpolarized electron-positron annihilation, is expressed as a combination of four parton fragmentation matrix elements. Since the inclusive Λ production we considered is theoretically the simplest, our study can be taken as a question to the operatability and practability of the first nonleading QCD factorization theorem in the transverse Λ polarization phenomena.

*Mailing address

Among the various spectacular phenomena relating to particle spins is the polarization perpendicular to the production plane of the inclusively detected hyperons in unpolarized fixed-target experiment [1,2]. Owing to the complexities of hadron-hadron processes, it occurs naturally to one that efforts should be made to study such transverse polarization phenomena in some simpler and cleaner circumstances. According to the QCD factorization theorem [3], the transverse Λ polarizations in different processes share a common set of parton fragmentation matrix elements (generalized parton fragmentation functions). Hopefully, one should be able to extract the data about these matrix elements in a simpler chosen process and then make predictions for the other complicated inclusive Λ productions. Such a philosophy is just like the practice that people measure the parton distribution functions in the deeply inelastic scattering and thereafter make predictions for other large-momentum-transfer processes.

Presumably, the inclusive Λ production by unpolarized e^-e^+ annihilation is the first candidate to supply us with information about transverse-spin-dependent fragmentation matrix elements. Recently, one of the authors [4] has pointed out that due to "final-state" interactions, the inclusively detected spin-half fermions in unpolarized e^-e^+ annihilation can be transversely polarized, characterized by a structure function $\tilde{F}$. Such a polarization is completely different from that caused by parity violation at the Z resonance [5], and easier to measure than the semi-inclusive $\Lambda\bar{\Lambda}$ production by unpolarized electron-positron annihilation [6]. Experimentally, the facilities such as the Beijing Electron-Positron Collider (BEPC), the B Factory under construction, and the purposed Tau-Charm Factories provide us a number of opportunities to access $\tilde{F}$ at various energy scales. In this paper, we present a QCD factorization approach to the photon fragmentation tensor for inclusive Λ production, with the Λ spin perpendicular to the production plane. Our results show that $\tilde{F}$ can be expressed as a combination of four parton fragmentation matrix elements. Unless parton fragmentation matrix elements happen to make contributions also in the form of such a combination in other inclusive Λ production processes, the hope is very faint to establish simple connections among the transverse Λ polarizations in different reactions.

Specifically, what we will consider is the process

$$e^-(p_1) + e^+(p_2) \to \gamma^*(q) \to \Lambda(p,s) + X, \tag{1}$$

whose invariant cross section can be written as

$$E\frac{d\sigma(s)}{d^3p} = \frac{2\alpha^2}{Q^5} L_{\mu\nu}(p_1,q)\hat{W}^{\mu\nu}(q,p,s). \tag{2}$$

At the Born Level, the leptonic tensor reads

$$\begin{aligned} L^{\mu\nu}(p_1,q) &= \frac{1}{2}\mathrm{Tr_D}[(\not{p}_1 + m_e)\gamma^\mu(\not{p}_2 - m_e)\gamma^\nu] \\ &= (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})\frac{q^2}{2} - 2(p_1^\mu - \frac{p_1\cdot q}{q^2}q^\mu)(p_1^\nu - \frac{p_1\cdot q}{q^2}q^\nu), \end{aligned} \tag{3}$$

where $\mathrm{Tr_D}$ signal the trace in the Dirac space. Concerning the hadronic tensor, it is defined as usual

$$\hat{W}_{\mu\nu}(q,p,s) = \frac{1}{4\pi}\sum_X \int d^4\xi \exp(iq\cdot\xi)\langle 0|j_\mu(0)|\Lambda(p,s),X\rangle\langle\Lambda(p,s),X)|j_\nu(\xi)|0\rangle, \tag{4}$$

where $\sum_X$ represents the summation over all the possible final states that contain the inclusive hyperon. The electromagnetic current is defined as $j^\mu = \sum_f e_f\bar{\psi}_f\gamma^\mu\psi_f$, with f being the quark flavor index and e_f being the electric charge of the quark in unit of the electron charge. We will consider the Λ production via the strange quark fragmentation so the flavor index will be suppressed wherever possible. Taking into account the constraints due to the gauge invariance, hermiticity and parity conservation, we have the following general Lorentz decomposition for $\hat{W}(q,p,s)$ [4]:

$$\begin{aligned} \hat{W}_{\mu\nu}(q,p,s) = \frac{1}{2}\Bigg[&(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\hat{F}_1(z_B,Q^2) + (p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)\frac{\hat{F}_2(z_B,Q^2)}{p\cdot q}\Bigg] \\ &+ iM\varepsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma\frac{\hat{g}_1(z_B,Q^2)}{p\cdot q} + iM\varepsilon_{\mu\nu\lambda\sigma}q^\lambda(s^\sigma - \frac{s\cdot q}{p\cdot q}p^\sigma)\frac{\hat{g}_2(z_B,Q^2)}{p\cdot q} \\ &+ M\left[(p_\mu - \frac{p\cdot q}{q^2}q_\mu)\varepsilon_{\nu\rho\tau\eta}p^\rho q^\tau s^\eta + (p_\nu - \frac{p\cdot q}{q^2}q_\nu)\varepsilon_{\mu\rho\tau\eta}p^\rho q^\tau s^\eta\right]\frac{\tilde{F}(z_B,Q^2)}{(p\cdot q)^2}, \end{aligned} \tag{5}$$

where $z_B \equiv 2(p\cdot q)/q^2$, $Q \equiv \sqrt{q^2}$, M is the mass of the Λ hyperon, and $\hat{F}_1$, $\hat{F}_2$, $\hat{g}_1$, $\hat{g}_2$, and $\tilde{F}$ are the scaling structure functions. Throughout the work, we normalize the spin vector in such a way that $s\cdot s = -1$ for the pure state of a spin-half particle.

By definition, the transverse Λ polarization reads

$$P_\Lambda = \frac{Ed\sigma(s_\uparrow)/d^3p - Ed\sigma(s_\downarrow)/d^3p}{Ed\sigma(s_\uparrow)/d^3p + Ed\sigma(s_\downarrow)/d^3p}, \tag{6}$$

where $\uparrow$ and $\downarrow$ represent the Λ spin parallel and antiparallel to the normal of the production plane, respectively. Substituting eqs. (3) and (5) into (2), we will have

$$P_\Lambda(z_B, Q^2, \theta) = \frac{4M\mathbf{p}^2 \sin\theta\cos\theta \tilde{F}(z_B, Q^2)}{E\left[2QE\hat{F}_1(z_B, Q^2) + \mathbf{p}^2\sin^2\theta\hat{F}_2(z_B, Q^2)\right]}, \tag{7}$$

where $\mathbf{p}$ is the momentum of the inclusively detected Λ particle in the center of mass frame and θ denotes its outgoing angle with respect to the electron beam direction.

Our coordinate system is specified by letting the $\hat{z}$ axis be along the outgoing direction of the inclusive hyperon and putting the $\hat{x}$–$\hat{z}$ plane in the production plane. We adopt the light-cone coordinates and parameterize the Λ momentum as

$$p^\mu = P^\mu + \frac{1}{2}M^2 n^\mu, \tag{8}$$

where

$$P^\mu = \frac{1}{\sqrt{2}}(\sqrt{M^2 + |\mathbf{p}|^2} + |\mathbf{p}|)(1^+, 0^-, 0_\perp), \tag{9}$$

$$n^\mu = \frac{\sqrt{2}}{M^2}(\sqrt{M^2 + |\mathbf{p}|^2} - |\mathbf{p}|)(0^+, 1^-, 0_\perp). \tag{10}$$

Obviously, P and n are light-like and they satisfy $P \cdot n = 1$. We will work in the frame in which $|\mathbf{p}|$ has a large value, then the plus components of the involved momenta are dominant.

As can be seen from eq. (7), the transverse Λ polarization is one-power suppressed, i.e., at twist three. Therefore, we will work to the level of twist three in factoring photon fragmentation tensor. We choose to work in the light-cone gauge specified by $n \cdot A = 0$, in which the color gauge invariance can be most easily recovered. Thus, our subjects are the diagrams shown in figs. 1, 2 and 3. The main difference between the two diagrams shown in fig. 2 and those in fig. 3 lies in that the gluon exchange takes place at the different energy scales. Since the QCD factorization is an expansion in terms of a hard scattering scale, it is necessary to distinguish these two cases. In fact, the diagrams in fig. 2 has already been discussed by Kane, Pumplin and Repko in their classic 1978 paper [7]. For completeness, we still include the contributions of these diagrams.

Two spin-independent structure functions, $\hat{F}_1$ and $\hat{F}_2$, have been thoroughly studied in ref. [8]. According to the generalized factorization theorem [9], both $\hat{F}_1$ and $\hat{F}_2$ are essentially at twist two, their first nonleading corrections being at twist four. Since we work up to twist three, it is enough to adopt the twist-two results for $\hat{F}_1$ and $\hat{F}_2$. That is, we simply utilize the parton model prescription:

$$\hat{F}_1(z_B) = \frac{2e_s^2}{z_B}\hat{f}_1(z_B), \tag{11}$$

$$\hat{F}_2(z_B) = -\frac{4e_s^2}{z_B^2}\hat{f}_1(z_B), \tag{12}$$

where $e_s = 1/3$ is the strange quark charge and $\hat{f}_1(z)$ is the spin-independent quark fragmentation. ($\hat{f}_1(z)$ is also labelled $D(z)$ in the literature, but we adopt Jaffe and Ji's notation [10,11] about quark fragmentation functions as much as possible.) Henceforth, we will concentrate ourselves on spin-dependent structure functions and simply suppress all the spin-independent terms in our presentation.

To the order at which we work,

$$\begin{aligned}\hat{W}^{\mu\nu}(q, p, s_\perp) = &\frac{1}{4\pi N}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr_D Tr_C}\left[\left(H^{\mu\nu}_{(1)}(q,k) + H^{\mu\nu}_{(2a)}(q,k) + H^{\mu\nu}_{(2b)}(q,k)\right)T(k,p,s_\perp)\right] \\ &+ \frac{1}{4\pi N}\int \frac{d^4k}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\mathrm{Tr_D Tr_C}\left[H^{\mu\nu\sigma}_{(3a)}(q,k,k_1)X'_\sigma(k,k_1,p,s_\perp)\right] \\ &+ \frac{1}{4\pi N}\int \frac{d^4k}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\mathrm{Tr_D Tr_C}\left[H^{\mu\nu\sigma}_{(3b)}(q,k,k_1)Y'_\sigma(k,k_1,p,s_\perp)\right],\end{aligned} \tag{13}$$

where $\mathrm{Tr_C}$ represents the trace in the color space, and

$$T_{\alpha\beta}(k,p,s_\perp) = \sum_X \int d^4\xi \exp(-ik\cdot\xi)\langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar{\psi}_\beta(\xi)|0\rangle, \tag{14}$$

$$X'^{\sigma}_{\alpha\beta}(k_1,k,p,s_\perp)=\sum_X\int d^4\xi d^4\xi_1\exp(-i(k-k_1)\cdot\xi_1-ik\cdot\xi)$$
$$\times\langle 0|(-g_s)A^\sigma(\xi_1)\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\xi)|0\rangle, \tag{15}$$

$$Y'^{\sigma}_{\alpha\beta}(k_1,k,p,s_\perp)=\sum_X\int d^4\xi d^4\xi_1\exp(-i(k_1-k)\cdot\xi-ik_1\cdot\xi_1)$$
$$\times\langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\xi_1)(-g_s)A^\sigma(\xi)|0\rangle. \tag{16}$$

On writing down eq. (13), we have utilized the shorthand A^σ for $A^\sigma_a T_a$, with T_a the fundamental representation of the color SU(3) group. For the diagrams in fig. 3, the color matrix along with a minus strong coupling, $-g_s T^a_{ij}$, has been isolated from the hard part into the fragmentation matrices. Throughout all, the color summation is assumed in our fragmentation matrices, so there is a color factor of $1/N$ ($N=3$) in our expressions of $\hat W^{\mu\nu}(q,p,s)$. Furthermore, we reserve the subscripts α and β for the Dirac indices.

As Ellis, Petronzio and Furmaski [12] have demonstrated in the case of the deeply inelastic scattering, the leading contributions associated with each diagram can be extracted by making an expansion about the components of the parton momenta that are collinear to the corresponding hadron momentum. Since we work up to twist three, it is needed to expand the lowest-order diagram to the first derivative term. For this purpose, we decompose the strange quark momentum into its plus, transverse, and minus components

$$k^\mu=\frac{1}{z}P^\mu+k^\mu_T+\frac{k^2-k_T^2}{2k\cdot n}n^\mu. \tag{17}$$

In the collinear expansion, the minus component is irrelevant. Therefore, we have

$$H_{\mu\nu}(q,k)=H_{\mu\nu}(q,P/z)+\frac{\partial H_{\mu\nu}(q,k)}{\partial k^\sigma}|_{k=P/z}(k-P/z)^\sigma+\cdots\ . \tag{18}$$

Concerning those diagrams in fig. 3, whose leading contributions come about at twist three, we can take their leading terms in the collinear expansion. As a result, we obtain

$$\hat W^{\mu\nu}(q,p,s_\perp)=\frac{1}{4\pi N}\int\frac{dz}{z}\mathrm{Tr_D}\mathrm{Tr_C}\left[\left(H^{\mu\nu}_{(1)}(q,P/z)+H^{\mu\nu}_{(2a)}(q,P/z)+H^{\mu\nu}_{(2b)}(q,P/z)\right)T(z,p,s_\perp)\right]$$
$$+\text{first derivative term of the lowest order diagram}$$
$$+\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz\mathrm{Tr_D}\mathrm{Tr_C}\left[H^{\mu\nu\sigma}_{(3a)}(q,P/z,P/z_1)X'_\sigma(z,z_1,p,s_\perp)\right]$$
$$+\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz\mathrm{Tr_D}\mathrm{Tr_C}\left[H^{\mu\nu\sigma}_{(3b)}(q,P/z,P/z_1)Y'_\sigma(z,z_1,p,s_\perp)\right], \tag{19}$$

where

$$T_{\alpha\beta}(z,p,s_\perp)=z\sum_X\int\frac{d\lambda}{2\pi}\exp(-i\lambda/z)\langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda n)|0\rangle, \tag{20}$$

$$X'^{\sigma}_{\alpha\beta}(z_1,z,p,s_\perp)=\sum_X\int\frac{d\lambda_1 d\lambda}{(2\pi)^2}\exp(-i\lambda_1(1/z-1/z_1)-i\lambda/z)$$
$$\times\langle 0|(-g_s)A^\sigma(\lambda_1 n)\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda n)|0\rangle, \tag{21}$$

$$Y'^{\sigma}_{\alpha\beta}(z_1,z,p,s_\perp)=\sum_X\int\frac{d\lambda_1 d\lambda}{(2\pi)^2}\exp(-i\lambda(1/z_1-1/z)-i\lambda_1/z_1)$$
$$\times\langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda_1 n)(-g_s)A^\sigma(\lambda n)|0\rangle. \tag{22}$$

Obviously, both $X'^{\sigma}_{\alpha\beta}(z_1,z,p,s_\perp)$ and $Y'^{\sigma}_{\alpha\beta}(z_1,z,p,s_\perp)$ are not gauge invariant objects. To recover the color gauge invariance, we have two options [13]: (1) Adding a derivative operator to $(-g_s)A^\sigma$ to form a covariant derivative; (2) Exploiting integration by parts to transform the gluonic field A^σ into the gluonic field tensor $F^{+\sigma}$.

Recalling that the uncut antiquark propagator in the hard part has the $i\varepsilon$ prescription, we need to distinguish two cases. In the case that the $i\varepsilon$ prescription is irrelevant, one can show by the Ward identity

$$\frac{\partial H_{\mu\nu}(q,k)}{\partial k^\sigma}|_{k=P/z} = H^{\mu\nu}_{(3a)\sigma}(q,P/z,P/z) = H^{\mu\nu}_{(3b)\sigma}(q,P/z,P/z), \tag{23}$$

then the contributions associated with the derivative term in eq. (19) can be combined with leading contributions of the two diagrams in fig. 3. The net effect of such a combination [14] is to replace $(-g_s)A^\sigma$ by the covariant derivative operator $D^\sigma = i\partial^\sigma - g_s A^\sigma$ in $X'^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$ and $Y'^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$.

As regard with the pole contribution in the higher-order diagram, the above strategy to reserve the gauge invariance is obviously not applicable. This stimulates us to consider the second option stated above, namely, transforming the gluonic field into the corresponding field tensor. Here we note that in the definitions of fragmentation matrices, an $i\epsilon$ prescription is implicit in the exponentials, which warranties the vanishing of matrix elements as $\lambda_1 \to 0$ or $\lambda \to 0$. Hence, we can write out

$$X'^\sigma_{\alpha\beta}(z_1,z,p,s_\perp) = \frac{-ig_s}{1/z - 1/z_1 + i\varepsilon} X^\sigma_{\alpha\beta}(z_1,z,p,s_\perp), \tag{24}$$

$$Y'^\sigma_{\alpha\beta}(z_1,z,p,s_\perp) = \frac{-ig_s}{1/z_1 - 1/z + i\varepsilon} Y^\sigma_{\alpha\beta}(z_1,z,p,s_\perp), \tag{25}$$

with

$$\begin{aligned} X^\sigma_{\alpha\beta}(z_1,z,p,s_\perp) = \sum_X \int \frac{d\lambda_1 d\lambda}{(2\pi)^2} &\exp(-i\lambda_1(1/z - 1/z_1) - i\lambda/z) \\ &\times \langle 0| n^- F^{+\sigma}(\lambda_1 n)\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda n)|0\rangle, \end{aligned} \tag{26}$$

$$\begin{aligned} Y^\sigma_{\alpha\beta}(z_1,z,p,s_\perp) = \sum_X \int \frac{d\lambda_1 d\lambda}{(2\pi)^2} &\exp(-i\lambda(1/z_1 - 1/z) - i\lambda_1/z_1) \\ &\times \langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda_1 n) n^- F^{+\sigma}(\lambda n)|0\rangle. \end{aligned} \tag{27}$$

However, the collinear expansion scheme is not a satisfactory procedure to extract nonleading twist contributions. As Qiu has demonstrated [14], the leading term in the collinear expansion contains as well the nonleading contributions. A remedy to the Ellis-Furmanski-Petronzio procedure is the "special" propagator technique invented by Qiu. Now we explain what Qiu's special propagator [14] is. Consider the quark propagator in fig. 3 that links the electromagnetic vertex to the quark-gluon one. From our parameterization (17), one can write

$$\frac{i\not k}{k^2 + i\varepsilon} = \frac{i\hat{\not k}}{k^2 + i\varepsilon} + \frac{i\not n}{2k\cdot n}. \tag{28}$$

where

$$\hat k^\mu = \frac{1}{z}P^\mu + k_T^\mu - \frac{k_T^2}{2k\cdot n}n^\mu \tag{29}$$

is the on-shell part of k^μ. Physically, the $in\cdot\gamma/(2k\cdot n)$ piece describes a "contact" interaction in the light-cone, so it should be included into the hard partonic interaction part. Since $in\cdot\gamma/(2k\cdot n)$ is a part of the propagator, Qiu termed it the special propagator. Graphically, it is labelled by adding a bar on the normal propagator. In our twist-three case, only *one* such special propagator along with the connected quark-gluon vertex needs to be invoked, either on the left-hand side or on the right-hand side of the final-state cut. In other words, twist-three contributions hidden in the leading term of the collinear expansion of the lowest-order diagram can be taken into account by including the two diagrams in fig. 4. Considering that the diagrams shown in fig. 4 can be obtained from the lowest-order diagram by pulling out a quark propagator and its connected quark-gluon vertex, it can also be stated that the non-contact part of its linking propagator should be discarded, because they have been included in $T_{\alpha\beta}(k,p,s_\perp)$. It should be stressed that Qiu's special propagator prescription can naturally reserve the gauge invariance for the hard part [14].

From the above discussions, we know that the formula for calculating the hadronic tensor reads

$$\begin{aligned} \hat W^{\mu\nu}(q,p,s_\perp) = \frac{1}{4\pi N}\int \frac{dz}{z} \mathrm{Tr_D Tr_C}&\left[\left(H^{\mu\nu}_{(1)}(q,P/z) + H^{\mu\nu}_{(2a)}(q,P/z) + H^{\mu\nu}_{(2b)}(q,P/z)\right) T(z,p,s_\perp)\right] \\ +\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz \mathrm{Tr_D Tr_C}&\left[\left(H^{\mu\nu\sigma}_{(3a)}(q,P/z,P/z_1) + H^{\mu\nu\sigma}_{(4a);\mathrm{spec}}(q,P/z)\right) U_\sigma(z,z_1,p,s_\perp)\right] \\ +\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz \mathrm{Tr_D Tr_C}&\left[\left(H^{\mu\nu\sigma}_{(3b)}(q,P/z,P/z_1) + H^{\mu\nu\sigma}_{(4b);\mathrm{spec}}(q,P/z)\right) V_\sigma(z,z_1,p,s_\perp)\right] \end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz\frac{-ig_s}{1/z-1/z_1+i\varepsilon}\\
&\times \mathrm{Tr_D Tr_C}\left[\left(H^{\mu\nu\sigma}_{(3a)}(q,P/z,P/z_1)+H^{\mu\nu\sigma}_{(4a);\mathrm{spec}}(q,P/z)\right)X_\sigma(z,z_1,p,s_\perp)\right]\\
&+\frac{1}{4\pi N}\int d(\frac{1}{z_1})dz\frac{-ig_s}{1/z_1-1/z+i\varepsilon}\\
&\times \mathrm{Tr_D Tr_C}\left[\left(H^{\mu\nu\sigma}_{(3b)}(q,P/z,P/z_1)+H^{\mu\nu\sigma}_{(4b);\mathrm{spec}}(q,P/z)\right)Y_\sigma(z,z_1,p,s_\perp)\right],
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
U^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&\sum_X\int\frac{d\lambda_1 d\lambda}{(2\pi)^2}\exp(-i\lambda_1(1/z-1/z_1)-i\lambda/z)\\
&\times\langle 0|D^\sigma(\lambda_1 n)\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda n)|0\rangle,
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
V^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&\sum_X\int\frac{d\lambda_1 d\lambda}{(2\pi)^2}\exp(-i\lambda(1/z_1-1/z)-i\lambda_1/z_1)\\
&\times\langle 0|\psi_\alpha(0)|\Lambda(p,s_\perp),X\rangle\langle\Lambda(p,s_\perp),X|\bar\psi_\beta(\lambda_1 n)\overleftarrow{D}^\sigma(\lambda n)|0\rangle.
\end{aligned}
\tag{32}
$$

with $\overleftarrow{D}^\sigma=-i\overleftarrow{\partial}^\sigma-g_sA^\sigma$.

Since the gluonic field can either be converted into a field tensor or combined with a derivative operator to form a covariant derivative operator, there seems to be a mixing between $X^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$ and $U^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$, and $Y^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$ and $V^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)$. In other words, one may question if the higher-twist contributions in eq. (30) are double-counted. Fortunately, this does not cause any problem in our present case. Our computational processes shows that the twist-three matrix elements with a covariant derivative operator are irrelevant of the pole part of the higher-order diagrams, whereas those with a gluonic field tensor make contributions only by virtue of the pole term.

To arrive at factorized expressions, we decompose the above three fragmentation matrices in the Dirac and Lorentz spaces. We suppress the spin-independent terms and go to the twist-three level for those spin-dependent terms. For $T_{\alpha\beta}(z,p,s_\perp)$, we have

$$
T_{\alpha\beta}(z,p,s_\perp)=\hat h_1(z)(\gamma_5\not{s}_\perp\not{P})_{\alpha\beta}+M\hat g_T(z)(\gamma_5\not{s}_\perp)_{\alpha\beta}+M\tilde g_T(z)\varepsilon^{\delta\eta\rho\tau}s_{\perp\eta}P_\rho n_\tau(\gamma_\delta)_{\alpha\beta}+\cdots.
\tag{33}
$$

As for the decomposition of our two-variable fragmentation matrices, we note that the gluonic field tensor is related to the covariant derivative operator via $F^{\mu\nu}=1/(ig_s)[D^\mu,D^\nu]$. Therefore, they have different behaviours under the time-reversal transformation. In discussing the spin asymmetry, it is more convenient to employ the adjoint parity-time-reversal transformation instead of the individual parity or time-reversal transformation. Under the adjoint transformation, the covariant derivative is even while the filed tensor behaves odd. As a result, we have the following decompositions

$$
\begin{aligned}
U^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&\frac{iM}{2z}\hat G_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{M}{2z}\hat G_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}\\
&+\frac{M}{2z}\tilde G_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{iM}{2z}\tilde G_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}+\cdots,
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
V^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&-\frac{iM}{2z}\hat G_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{M}{2z}\hat G_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}\\
&+\frac{M}{2z}\tilde G_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}-\frac{iM}{2z}\tilde G_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}+\cdots,
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
X^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&\frac{M}{4\pi z}\hat H_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{iM}{4\pi z}\hat H_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}\\
&+\frac{iM}{4\pi z}\tilde H_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{M}{4\pi z}\tilde H_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}+\cdots,
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
Y^\sigma_{\alpha\beta}(z_1,z,p,s_\perp)=&\frac{M}{4\pi z}\hat H_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}-\frac{iM}{4\pi z}\hat H_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}\\
&-\frac{iM}{4\pi z}\tilde H_1(z_1,z)\varepsilon_{\sigma\rho\tau\eta}s^\rho_\perp P^\tau n^\eta\not{P}_{\alpha\beta}+\frac{M}{4\pi z}\tilde H_2(z_1,z)s^\sigma_\perp(\gamma_5\not{P})_{\alpha\beta}+\cdots.
\end{aligned}
\tag{37}
$$

In our work, we use a tilde to signal those matrix elements that arise from the hadronic final-state interactions. The definitions of the matrix elements can be easily obtained by projecting the corresponding matrix elements with the appropriate projectors. $\hat{h}_1(z)$, $\hat{g}_T(z)$, and $\tilde{g}_T(z)$ (labelled $\hat{g}_{\bar{T}}(z)$ by Jaffe and Ji) are Jaffe and Ji's single-variable quark fragmentation functions [10,11]. $\hat{G}_1(z_1,z)$ and $\hat{G}_2(z_1,z)$ were also introduced first in ref. [11], but the other six two-variable twist-three fragmentation matrix elements appear first time in this paper. These eight constitute a complete set of transverse-spin-dependent, two-variable, and twist-three parton fragmentation matrix elements.

Substituting eqs. (33)–(37) into (30), we obtain after some algebra

$$\begin{aligned}\hat{W}_{\mu\nu}(q,p,s_\perp) = & \frac{ie_s^2}{z_0^2(P\cdot q)}\left[z_0 m_s \hat{h}_1(z_0) + M\hat{g}_T(z_0)\right]\varepsilon_{\mu\nu\tau\rho}q^\tau s_\perp^\rho. \\ & + \frac{e_s^2}{4z_0^2(P\cdot q)^2}\left[\frac{N^2-1}{N}\alpha_s m_s \hat{h}_1(z_0) - \frac{1}{2}Mg_s z_0\left(\hat{H}_1(z_0,z_0)-\hat{H}_2(z_0,z_0)\right) - M\tilde{g}_T(z_0)\right] \\ & \times\left[(p_\mu - \frac{p\cdot q}{q^2}q_\mu)\varepsilon_{\nu\rho\tau\eta}p^\rho q^\tau s_\perp^\eta + (p_\nu - \frac{p\cdot q}{q^2}q_\nu)\varepsilon_{\mu\rho\tau\eta}p^\rho q^\tau s_\perp^\eta\right],\end{aligned} \tag{38}$$

where $z_0 \equiv 2(P\cdot q)/q^2$. According to eqs. (8-10), z_0 is same as z_B up to a piece of twist-four effects.

Here some notes are in order.

(1). To arrive at this expression with manifest electromagnetic gauge invariance, we have employed the following projection relations:

$$\int d(\frac{1}{z_1})[\hat{G}_1(z_1,z)+\hat{G}_2(z_1,z)] = -\frac{1}{z}\hat{g}_T(z) + \frac{m_s}{M}\hat{h}_1(z), \tag{39}$$

$$\int d(\frac{1}{z_1})[\tilde{G}_1(z_1,z)+\tilde{G}_2(z_1,z)] = -\frac{1}{z}\tilde{g}_T(z). \tag{40}$$

The relations of this type was first identified by Effremov and Teryaev [15] in their studies of single transverse spin asymmetries. Equation (39), with its quark mass term neglected, has ever appeared in ref. [11]. In this quark fragmentation version, we included the effects of the quark mass associated with chiral-odd, twist-two quark fragmentation function $\hat{h}_1(z)$. However, there is no quark mass term in eq. (40), because there is no final-state-interaction-caused quark fragmentation function at twist two.

(2). The diagrams in fig. 2 can be taken as vertex corrections to the lowest-order diagram in fig. 1. In fact, their contributions have been discussed in the framework of the quark parton model in ref. [7]. At the twist at which we work, the contributions of those two diagrams in fig. 2 can be included by replacing the fundamental electromagnetic vertex γ^μ in fig. 1 with the effective vertex operator

$$\Gamma^\mu = \gamma^\mu\left[1+f(q^2)\right] + \frac{1}{2m_s}\sigma^{\mu\tau}q_\tau g(q^2), \tag{41}$$

or similarly for γ^ν. For the transverse polarization requires flipping the chirality, only the Dirac tensor part is relevant. By considering the imaginary part of the decay amplitudes for $\gamma^* \to s\bar{s}$, one can obtain [16],

$$g(q^2) = \frac{N^2-1}{2}\frac{e_s^2\alpha_s m_s^2}{\sqrt{q^2(q^2-4m_s^2)}}. \tag{42}$$

Obviously, the contributions of the vertex corrections are insignificant for they vanish in the scaling limit.

(3). Our factorization result, eq. (38) is complete to the $O(Q^{-1})$. Therefore, we can make substitutions $P\to p$ and $z_0 \to z_B$, which includes some effects at two higher twist. Then, we can confront our result with the general decomposition of $\hat{W}^{\mu\nu}(q,p,s)$, eq. (5). As a result,

$$\hat{g}_1(x_B) + \hat{g}_2(x_B) = \frac{1}{9z_B^2}\left[\frac{m_s}{M}z_B\hat{h}_1(z_B) + \hat{g}_T(z_B)\right], \tag{43}$$

$$\tilde{F}(z_B) = \frac{1}{36z_B^2}\left[\frac{N^2-1}{N}\frac{m_s}{M}\alpha_s\hat{h}_1(z_0) - \frac{1}{2}g_s z_B\left(\hat{H}_1(z_B,z_B)-\hat{H}_2(z_B,z_B)\right) - \tilde{g}_T(z_B)\right]. \tag{44}$$

In summary, our study shows that structure function $\tilde{F}$ is related to a combination of four parton fragmentation matrix elements, even we assume that the Λ particle production is predominantly via the strange quark fragmentation. At present, there is no reliable way to calculate them from the first principle–QCD. Actually, the predictive power of QCD factorization lies in that different processes are related by a common set of nonperturbative matrix elements. Although $\tilde{F}$ can be measured

by experiments, it cannot be expected to extract easily information about individual fragmentation matrix elements. If one can justify that one of them plays a leading role in certain phase space, however, it will become a possible practice. At the time being, such justifications is obviously beyond reach. At first sight, one may argue that the contribution associated with $\hat{h}_1(z)$, which characterizes the chiral symmetry breaking effects due to the strang quark mass, is negligible. But it is still unknown to what extent the other three matrix elements, $\tilde{g}_T(z)$, $\hat{H}_1(z, z)$ and $\hat{H}_2(z, z)$, are important. Therefore, this work exemplified the potential difficulties in applying the generalized QCD factorization theorem to the transverse Λ polarization problem, that is, the number of the relevant fragmentation matrix elements is so large that one cannot find a simple standard process to normalize the others. Nevertheless, if the parton fragmentation matrix elements happen to contribute in the combinational form that we obtained, it is still possible to establish a simple connection between the various transverse Λ polarization phenomena. Although such a hope is very faint, we feel it is still desirable to work on a factorized expression for transverse Λ polarization in hadron-hadron process by applying the scheme outlined in this paper.

One of the authors (W. L.) thanks X. Ji for correspondences as well as for a prepublication copy of ref. [11].

---

[1] B. Lundberg et al., Phys. Rev. D 40 (1989) 3557; and the papers cited therein.
[2] L.G. Pondrom, Phys. Rep. 122, (1985) 57; and the papers cited therein.
[3] J. Collins, D.E. Soper and G. Sterman, in: Perturbative quantum chromodynamics, ed. A.H. Mueller (World Scientific, Singapore, 1989).
[4] W. Lu, Phys. Rev. D 51 (1995) 5305.
[5] M. Burkardt and R.L. Jaffe, Phys. Rev. Lett. 70, (1993) 2537.
[6] K. Chen, G.R. Goldstein, R. L. Jaffe and X. Ji, Nucl. Phys. B445, (1995) 380.
[7] G.L. Kane, J. Pumpkin, and W. Repko, Phys. Rev. Lett. 41, (1978) 1689.
[8] P. Nason and B.R. Webber, Nucl. Phys. B 421 (1994) 473.
[9] J. Qiu and G. Sterman, in: Polarized collider workshop, ed. J.C. Collins, S.F. Heppelman and R.W. Robinett(American Institute of Physics, New York, 1990); Nucl. Phys. B 353 (1991) 137.
[10] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 71 (1993) 2547.
[11] X. Ji, Phys. Rev. D 49 (1994) 114.
[12] R.K. Ellis, W. Furmanski and R. Petrozio, Nucl. Phys. B 207 (1982) 1; B 212 (1983) 29.
[13] X. Ji, Phys. Lett. B 289, (1992) 137.
[14] J. Qiu, Phys. Rev. D 42 (1990) 30.
[15] A.V. Effremov and O.V. Teryaev, Phys. Lett. B 150, (1985) 383.
[16] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, *Relativistic Quantum Theory* (Pergamon Press, Oxford, 1971).

Figure Captions

1. The lowest-order cut diagram for the inclusive Λ hyperon production by a time-like photon.
2. The cut diagram for the inclusive hyperon production by a time-like photon with one gluon correlation before the quark fragmentation.
3. The cut diagram for the inclusive hyperon production by a time-like photon with one gluon correlation at the quark fragmentation stage.
4. The cut diagram for the inclusive Λ hyperon production by a time-like photon with one gluon radiation in the quark fragmentation. One "special" propagator is pulled down into the hard partonic interaction part, either (a) on the left-hand side or (b) on the right-hand side of the final-state cut.

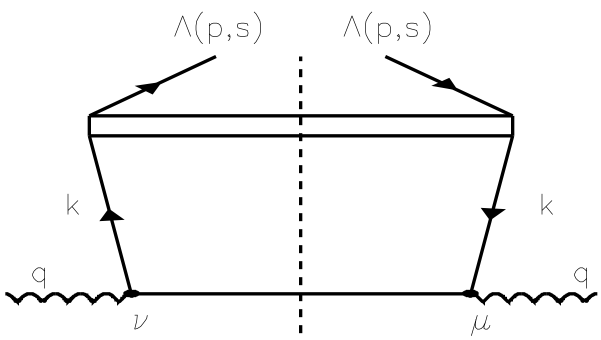

Fig. 1

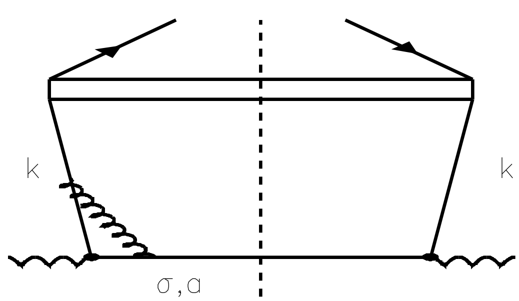

Fig. 2(a)

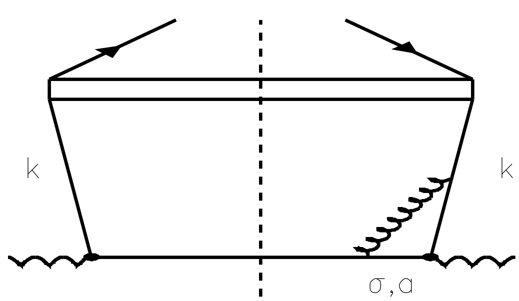

Fig. 2(b)

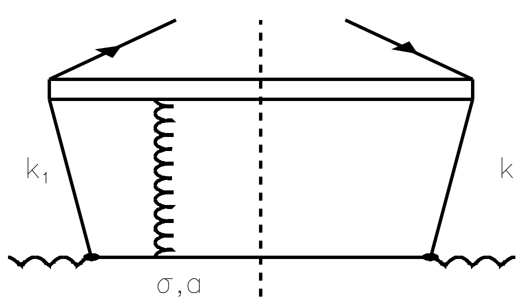

Fig. 3(a)

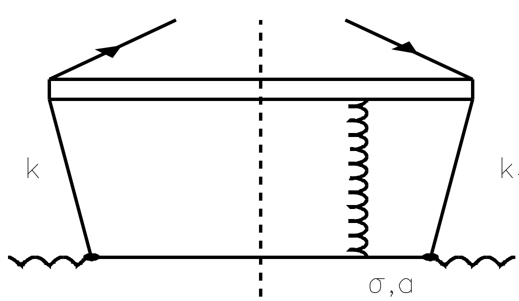

Fig. 3(b)

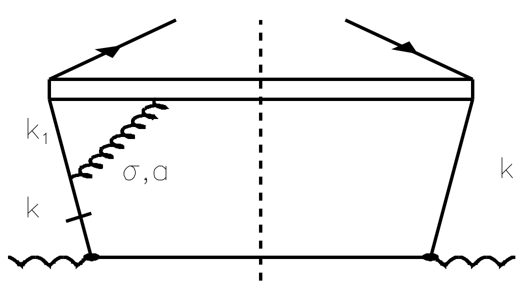

Fig. 4(a)

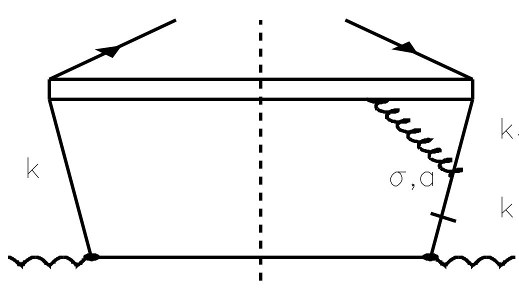

Fig. 4(b)